

Theory and Phenomenology of μ in M theory

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We consider a solution to the μ -problem within M theory on a G_2 -manifold. Our study is based upon the discrete symmetry proposed by Witten that forbids the μ -term and solves the doublet-triplet splitting problem. We point out that the symmetry must be broken by moduli stabilization, describing in detail how this can occur. The μ -term is generated via Kahler interactions after strong dynamics in the hidden sector generate a potential which stabilizes all moduli and breaks supersymmetry with $m_{3/2} \sim 20 - 30$ TeV. We show that μ is suppressed relative to the gravitino mass, by higher dimensional operators, $\mu \sim 0.1 m_{3/2} \sim 2 - 3$ TeV. This necessarily gives a Higgsino component to the (mostly Wino) LSP, and a small but non-negligible LSP-nucleon scattering cross-section. The maximum, spin-independent cross-sections are not within reach of the current XENON100 experiment, but are within reach of upcoming runs and upgrades.

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1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1], the only low energy supersymmetric parameter with mass dimension one is the μ parameter. Through the μ -term, $W \supset \mu H_u H_d$, it gives mass to Higgsinos and also generates scalar potential couplings for Higgs fields. The size of μ plays an important role in phenomenology. In particular, it affects properties of potential dark matter particles. LEP searches for the charged Higgsino require $\mu \gtrsim 100$ GeV, while arguments against fine tuning of the mass of the Z-boson suggest that μ should not be too large. On the other hand, one might expect, with ignorance of the high scale theory, that $\mu \sim m_{GUT}$, the natural UV cutoff. Solving the μ -problem [2] presumably requires an understanding of the fundamental theory that generates the scale of the μ parameter. Thus the μ -problem is exceptionally important—a high scale theory cannot be qualitatively complete without addressing it, and its solution will have significant implications for dark matter, Higgs physics, and fine-tuning issues.

The most promising framework for a complete fundamental theory that incorporates low energy supersymmetry is string theory. Within string theory, many explanations for the small value of μ have been proposed. In most scenarios the μ -term is forbidden at the high scale. Then, it is somehow dynamically generated at a lower scale. In many cases, the μ -term is forbidden by a continuous or discrete symmetry, which is spontaneously broken at a smaller, dynamically generated scale ($\ll m_{GUT}$), and perhaps related to supersymmetry breaking [3, 4]. Some examples of the above include NMSSM scenarios [5–10] and approximate R -symmetric models [11, 12]. Others scenarios have the μ -term forbidden by stringy selection rules, and are broken by non-perturbative instanton effects that produce exponentially suppressed mass scales [13–16].

It has long been suspected that the MSSM unifies the strong and electroweak forces [17] into a single $SU(5)$ grand unified group. Each family of quarks and leptons are organized into a $\mathbf{10} \oplus \bar{\mathbf{5}}$ representation of $SU(5)$. The remaining MSSM fields, the Higgs doublets, do not form a complete $SU(5)$ representation. Minimally, the Higgs doublets can be assigned to a $\mathbf{5} \oplus \bar{\mathbf{5}}$ representation, but require the introduction of a pair of Higgs color triplets. The Higgs triplets can mediate baryon and lepton violating processes, and thus should be very heavy, $m_T \gtrsim 10^{14}$ GeV, to avoid rapid proton decay [18]. Additionally, they should be heavy to ensure gauge coupling unification in the minimal model. If Higgs triplet masses are very heavy, then an $SU(5)$ symmetric theory would require that the Higgs doublets masses be the same as the triplet mass, $\mu = m_T \sim M_{GUT}$, but it was just argued that is this a factor 10^{13} too large. A string theoretic solution to the μ -problem is inevitably related to the solution of the doublet-triplet problem of grand unified theories.

Therefore, it is paramount that the symmetry that protects the μ -term not forbid the triplet masses if both problems are to be solved. This restriction leads to an elegant, perhaps unique solution to the μ -problem in M theory; the symmetry which protects μ from being generated at the unification scale was originally proposed by Witten [19]. Although Witten did not discuss how this symmetry would be broken, we argue that the symmetry would—indeed must—be broken by moduli stabilization. Then by including the mechanism for stabilizing the moduli proposed in [20], we will show that $\mu \sim 0.1 m_{3/2}$. Finally, the implications for dark matter discovery are discussed, where we conclude that the XENON100 experiment should not observe a dark matter signal, but may do so in its next upgrade (Figure 3).

2 M theory

2.1 Matter and Gauge Theory

In M theory compactified on a G_2 manifold, ADE gauge symmetries ($SU(n)$, $SO(2n)$ and E_6 , E_7 , E_8) are localized along three dimensional submanifolds of orbifold singularities [21, 22]. Chi-

ral matter, charged under the ADE gauge theory, is localized at conical singularities in the seven dimensional G_2 manifold, at points where the ADE singularity is enhanced [23–25]. Matter will additionally be charged under the $U(1)$ symmetry, corresponding to the vanishing 2-cycle that enhances the singularity. Hence, all chiral matter will be charged under at least one $U(1)$ symmetry. Bi-fundamental matter, charged under two non-Abelian gauge groups, is also possible, but will not be considered here.

As argued by [26], the additional $U(1)$ symmetries are never anomalous. Therefore, there is no Green-Schwarz mechanism [27] needed for anomaly cancellation, and GUT-scale FI D -terms are not present in the theory. This will be important later, since it removes a possibility for generating large scalar vacuum expectation values (vevs) for charged matter fields.

Two gauge theories will generically only have precisely the same size gauge coupling if they arise from the same orbifold singularities. Therefore, if gauge coupling unification is to be motivated theoretically, and not an approximation or accident, the gauge group of the ADE singularity should be a simple group containing the Standard Model gauge group, which we will take (for simplicity) to be $SU(5)$. Any larger group containing $SU(5)$ will give results similar to those we find below. To obtain the Standard Model gauge group, $SU(5)$ needs to be broken. Perhaps the 4D gauge symmetry can be broken spontaneously, but only representations smaller than the adjoint are realizable in M theory—the 10 and 5 representations (and their conjugates) in $SU(5)$. This leaves only “flipped $SU(5)$ ” [28–30] as a possible mechanism to break the GUT group and solve doublet-triplet splitting. Given the difficulty in constructing a realistic flipped $SU(5)$ model [31], it will not be considered here. The remaining possibility is to break the higher dimensional gauge theory by Wilson lines and will be discussed below.

2.2 Moduli Stabilization

In the mid-80’s it was realized that, classically, string vacua contain a plethora of moduli fields. The standard lore was that, after supersymmetry breaking, the moduli fields would obtain masses and appropriate vacuum expectation values. Part of this lore was also the idea that strong dynamics in a hidden sector would be responsible for breaking supersymmetry at, or around, the TeV scale. Though some progress was made, it was not until recently that it has been clearly demonstrated that these ideas can be completely realized in string/ M theory: in M theory compactified on a G_2 -manifold (without fluxes) strong gauge dynamics can generate a potential which stabilizes all moduli and breaks supersymmetry at a hierarchically small scale [20, 32]. These vacua will be the starting point for our considerations.

In these vacua, the gravitino mass (and therefore also the moduli masses [33]) $m_{3/2} \sim \frac{\Lambda^3}{m_{pl}^2}$, where Λ is the strong coupling scale of the hidden sector gauge interaction. This is parametrically

of order $\Lambda \sim e^{-2\pi/(\alpha_h b)} m_{pl}$, where α_h is the coupling constant of the hidden sector and b is a beta-function coefficient. The vacuum expectation values of the moduli fields are also determined in terms of α_h : Roughly speaking, one has:

$$\langle s^A \rangle \sim 1/\alpha_h \quad (2.1)$$

where the modulus here is dimensionless and not yet canonically normalized. The physical meaning of the vevs of s^A is that it characterizes the volumes in eleven dimensional units of 3-cycles in the extra dimensions, e.g., the 3-cycle that supports the hidden sector gauge group. Thus, self-consistently when the hidden sector is weakly coupled in the UV, the moduli are stabilized at large enough volumes in order to trust the supergravity potential which only makes sense in this regime. In general, the rough formula exhibits the scaling with α_h and, numerically the moduli vevs in the vacua considered thus far range from about $1 \leq s^A \leq 5/\alpha_h$.

In order to incorporate the moduli vevs into the effective field theory in an M theory vacuum, we have to consider the normalized dimensionful vevs which appear in the Einstein frame supergravity Lagrangian. For obtaining the normalization it suffices to consider the moduli kinetic terms alone:

$$\mathcal{L} \supset m_{pl}^2 \frac{1}{2} g_{AB} \partial_\mu s^A \partial^\mu s^B \quad (2.2)$$

where s^A are the dimensionless moduli described above and g_{AB} is the (Kahler) metric on the moduli space. From the fact that the extra dimensions have holonomy G_2 , it follows that each component of g_{AB} is homogeneous of degree *minus* two in the moduli fields

$$g_{AB} = \partial_A \partial_B K = \partial_A \partial_B (-3 \ln V_7 + \dots) \quad (2.3)$$

because the volume of X , V_7 is homogeneous of degree $7/3$.

For isotropic G_2 -manifolds, i.e. those which receive similar order contributions to their volume from each of the N moduli, studying examples shows that, not only is the metric of order $\frac{1}{s^2}$, but also of order $1/N$:

$$g \sim \frac{1}{N} \frac{1}{(s^A)^2} \quad (2.4)$$

Therefore in a given vacuum the order of magnitude of the entries of g_{AB} are

$$g \sim \frac{\alpha_h^2}{N} \quad (2.5)$$

Therefore, a dimensionless modulus vev of order $1/\alpha_h$ translates into a properly normalized dimensionful vev

$$\langle \hat{s}^A \rangle \sim \frac{1}{\sqrt{N}} \sim 0.1 m_{pl} \quad (2.6)$$

for $N \sim 100$, which is a typical expectation for the number of moduli [34]¹.

This can lead to a suppression of the effective couplings which generate the μ -term, once the symmetry forbidding μ is broken. More precise calculations of the moduli vevs can be found in [32, 35]. Clearly, however, a G_2 -manifold with less than ten or so moduli will not have suppressed, normalized moduli vevs; such cases are presumably unlikely candidates for G_2 -manifolds with realistic particle spectra and will not be considered further.

We briefly also discuss the spectrum of Beyond Standard Model (BSM) particles which arise from the M theory vacuum. Classically, it is well known that string/ M theory has no vacuum with a positive cosmological constant (de Sitter minimum). From the effective field theory point of view, this is the statement that moduli fields tend to have potentials which, in the classical limit have no de Sitter minimum. If we now consider quantum corrections to the moduli potential, which *only* involve the moduli fields – if they are computed in a perturbative regime – they tend to be small and hence are unlikely to generate de Sitter vacua. Positive, larger sources of vacuum energy must therefore arise from other, non-moduli fields. This is indeed the case in the M theory vacua described in [32]. Here the dominant contribution to the vacuum energy arises from a *matter* field in the hidden sector (where it can be shown that, without the matter field, no de Sitter vacuum exists). This is important for the following reasons.

Adopting supersymmetric terminology, this suggests that the fields with the dominant F -terms are not moduli. Hence, the moduli F -terms are suppressed relative to the dominant contribution (in fact, in M theory the suppression is of order α_h). This affects the spectrum of BSM particles. In string/ M theory, gaugino masses are generated through F -terms of moduli vevs (because the gauge coupling function is a superfield containing volume moduli). Hence, at leading order these will be suppressed relative to, say, scalar masses which receive order $m_{3/2}$ contributions from all F -terms in the absence of accidental symmetries. Therefore, in the G_2 -MSSM (and presumably other classes of string vacua) the scalar superpartners and moduli fields will have masses of order $m_{3/2}$ whereas the gaugino's will have masses which are suppressed; in fact in the G_2 -MSSM the gaugino masses at the GUT scale are at least two orders of magnitude below $m_{3/2}$. This is what makes the anomaly mediated contributions to gaugino masses relevant to the G_2 -MSSM and also why the models often contain a Wino LSP [35]. Important for our considerations below will be the fact that the suppression of the gaugino masses is greater than the suppression of moduli vevs discussed above by one order of magnitude (at the GUT scale), at least for G_2 -manifolds with less than $O(10^4)$ moduli.

¹Presumably, N is of the same order as the number of renormalizable coupling constants of the effective low energy theory.

2.3 Geometric Symmetries and Moduli Transformations

Compact, Ricci-flat manifolds with finite fundamental groups, such as manifolds with holonomy G_2 or $SU(3)$ can not have continuous symmetries. They can, however, have *discrete* symmetries. Witten was considering just such a discrete symmetry (G) of a G_2 -manifold when he proposed the symmetry which prevents μ . Assuming the simplest possibility of an Abelian discrete symmetry, let us consider $G = \mathbf{Z}_N$, which acts on X :

$$\mathbf{Z}_N : X \longrightarrow X \quad (2.7)$$

As a result of this, it will also act naturally on the fields on X . In particular \mathbf{Z}_N will act on the set of harmonic forms on X . Our interest here is $H^3(X, \mathbf{R})$ the set of harmonic 3-forms on X , since this locally represents the moduli space of G_2 -manifolds. A G_2 -manifold, with moduli at a point $\langle s^S \rangle = s_0^A$ is determined by a harmonic (locally) G_2 invariant 3-form φ as

$$\varphi = \sum s_0^A \beta_A \quad (2.8)$$

where β_A are a basis for $H^3(X, \mathbf{R})$. If the point s_0^A is such that \mathbf{Z}_N is a symmetry, then φ will be invariant under \mathbf{Z}_N , because invariance of φ is equivalent to invariance of the metric. The three-forms β_A transform in a representation of \mathbf{Z}_N , which is a real representation because the 3-forms are real on a G_2 -manifold. Hence,

$$\mathbf{Z}_N : \beta_A \rightarrow M_A^B \beta_B \quad (2.9)$$

where M is defined by this equation.

The fact that the particular G_2 -manifold, characterized by the particular point in moduli space s_0^A , is \mathbf{Z}_N -invariant is simply the statement that:

$$s_0^B M_A^B = s_0^A \quad (2.10)$$

i.e., the s_0^A are an eigenvector of M with unit eigenvalue. Clearly, this will not be true for a generic vector s^A ; hence, for a generic point in the moduli space, the entire \mathbf{Z}_N symmetry will be broken. Since the representation of \mathbf{Z}_N defined by the matrix M is real, it must be the sum of a complex representation plus its conjugate. Thus, the basis β_B can be chosen such that the complex representation is spanned by *complex* linear combinations of moduli fields. For instance, there might be a linear combination

$$S = \hat{s}^1 + i\hat{s}^2 \quad (2.11)$$

which we choose to write in-terms of the dimensionful fields (\hat{s}), that transforms as

$$S \rightarrow e^{2\pi i/N} S. \quad (2.12)$$

Since we usually consider complex representations of discrete symmetries acting on the matter fields in effective field theories, it will be precisely the linear combinations of moduli (those in the form (2.11)) which span $\mathbf{r}_\mathbb{C}$ which will appear in the "symmetry breaking sector" of the effective Lagrangian. In other words, the moduli will appear in complex linear combinations such as (2.11) in the Kahler potential operators containing other fields that transform under the \mathbf{Z}_N . Note that in (2.11) we are abusing notation in the sense that the " i " which appears is in general an N -by- N matrix whose square is minus the identity.

3 Witten's Solution

In heterotic and type-II string theories doublet-triplet splitting is often solved via orbifold compactifications [36, 37]. In these theories, higher (space-time) dimensional gauge symmetries are broken by the Wilson lines in an orbifold compactification, while the Kaluza-Klein zero mode Higgs triplets are absent due to non-trivial transformations under the orbifold symmetry. On the contrary, matter fields in M theory are co-dimension 7, that is, the fields live only in four dimensions, and are not zero modes of a KK tower of fields, so this solution to the μ -problem will not work. Other possibilities, such as NMSSM realizations or string instanton effects, will also not work since the symmetry that forbids μ (a $U(1)$ or stringy selection rules) would also forbid the triplet mass, thus spoiling doublet-triplet splitting.

One may also consider the possibility that a discrete R -symmetry can forbid the μ -term while solving doublet-triplet splitting. Requiring the symmetry to be anomaly free, and that it commutes with the gauge theory can lead to a unique symmetry [38]. However, this symmetry will also forbid the triplet mass and spoil doublet triplet splitting unless the triplets are absent from the four dimensional theory. For most string theories, this can be accomplished by a Wilson line in the higher dimensional theory, but in M theory, this is not possible since matter only exists in four dimensions.

Therefore, an alternative approach is needed to solve doublet-triplet splitting in M theory. The only known possibility, originally discussed by Witten, is to construct a discrete \mathbf{Z}_N symmetry of the geometry, that will act on both matter fields and moduli-fields. When combined with a discrete Wilson line that breaks $SU(5)$, this symmetry need not commute with the $SU(5)$, thus allowing components of a single $SU(5)$ representation to have different \mathbf{Z}_N charges. Since the above arguments demonstrate that there must be a symmetry that acts differently on doublets and triplets, so far this is the only approach known to work, and maybe be the only solution.

The minimal $SU(5)$ matter content contains three generations of matter descending from three copies of $\mathbf{10}_M \oplus \bar{\mathbf{5}}_M$. There is also a $\mathbf{5}_H \oplus \bar{\mathbf{5}}_H$ pair containing the MSSM Higgs doublets, $H_u \oplus H_d$, and a vector-like pair of Higgs triplets, $T_u \oplus T_d$. Here a doublet and a triplet from one of the Higgs

representations can transform differently under the \mathbf{Z}_N symmetry group. Without loss of generality or phenomenology, this field is taken to be the $\bar{\mathbf{5}}_H$ field, with the following charges for the fields

Field	\mathbf{Z}_N
$\mathbf{10}_M$	η^σ
$\bar{\mathbf{5}}_M$	η^τ
$\mathbf{5}_H$ T_u	η^α
$\mathbf{5}_H$ H_u	η^α
$\bar{\mathbf{5}}_H$ T_d	η^γ
$\bar{\mathbf{5}}_H$ H_d	η^δ

(3.1)

where $\eta \equiv e^{2\pi i/N}$.

These charges are constrained by the requirement that the \mathbf{Z}_N -symmetry does not forbid necessary terms in the superpotential, such as Yukawa couplings, Majorana neutrino masses, and the Higgs triplet masses

Coupling		Constraint
Up Yukawa Coupling	$\mathbf{10}_M \mathbf{10}_M H_u$	$2\sigma + \alpha = 0 \pmod N$
Down Yukawa Coupling	$\mathbf{10}_M \bar{\mathbf{5}}_M H_d$	$\sigma + \tau + \delta = 0 \pmod N$
Majorana Neutrino Masses	$H_d H_d \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M$	$2\alpha + 2\tau = 0 \pmod N$
Triplet Masses	$T_u T_d$	$\alpha + \gamma = 0 \pmod N.$

(3.2)

The solution to these equations are

$$\begin{aligned}
\alpha &= -2\sigma \\
\gamma &= 2\sigma \\
\delta &= -3\sigma + N/2 \\
\tau &= 2\sigma + N/2 \\
\sigma &= \sigma.
\end{aligned}
\tag{3.3}$$

Inherently, the \mathbf{Z}_N should forbid the μ -term, and if possible, other dangerous terms, such as dimension-5 proton decay operators and dimensions 3 and 4 R-parity violation.

Coupling		Constraint
μ - term	$H_d H_u$	$-5\sigma + N/2 \neq 0 \pmod N$
D-5 Proton Decay	$\mathbf{10}_M \mathbf{10}_M \mathbf{10}_M \bar{\mathbf{5}}_M$	$5\sigma - N/2 \neq 0 \pmod N$
D-3 R-Parity	$\mathbf{5}_H \bar{\mathbf{5}}_M$	$N/2 \neq 0 \pmod N$
D-4 R-Parity	$\mathbf{10}_M \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M$	$5\sigma \neq 0 \pmod N.$

(3.4)

Doublet-triplet splitting occurs if $5\sigma \neq N/2 \pmod N$. If one only wants to solve doublet-triplet splitting while forbidding the μ -term, then there is a solution for $N = 2$ and $\sigma = 1$. Forbidding all the dangerous operators above can be accomplished with a \mathbf{Z}_4 symmetry.

An essential point is that the existing bounds coming from the LEP experiments assert that the masses of charged Higgsinos are at least 100 GeV, hence an effective μ -term must be generated. In our context here this implies that the \mathbf{Z}_N symmetry must be broken, an aspect not discussed in [19]. This symmetry breaking is the subject of the next section.

4 Generating μ

As discussed in above, the \mathbf{Z}_N symmetry is a geometric symmetry of the internal G_2 manifold, under which the moduli are charged. The G_2 moduli [20] reside in chiral supermultiplets whose complex scalar components,

$$z^j = t^j + i s^j, \quad (4.1)$$

are formed from the geometric moduli of the manifold², s_i , and axionic components of the three-form C -field, t_i . We expect the moduli to break the discrete symmetry just below Planck scale when their vevs are stabilized [32, 35] (see Section (2.2)),

$$\langle \hat{s}_i \rangle \sim 0.1 m_p. \quad (4.2)$$

Likewise, the moduli F terms are expected to give gaugino masses in the usual way, so that

$$\langle F_{z^i} \rangle \simeq m_{1/2} m_p. \quad (4.3)$$

where $m_{1/2}$ is the tree level gaugino mass at the GUT scale. The axion shift symmetries $t_i \rightarrow t_i + a_i$ require that only imaginary parts of the moduli appear in perturbative interactions. The superpotential, being holomorphic in the fields, will not contain polynomial terms that explicitly depend on the moduli. The μ -term can then only be generated via Kahler interactions when supersymmetry is broken via a Giudice-Massiero like mechanism [39], i.e., from Kahler potential couplings quadratic in the Higgs fields.

To understand the size of μ (and $B\mu$) we first find a combination of moduli fields (or product of moduli fields), invariant under the axion symmetries, that transform under (a complex representation of) \mathbf{Z}_N with charge $5\sigma - N/2$

$$S^1 = \hat{s}^i + i \hat{s}^j \quad (4.4)$$

and another with charge $-5\sigma - N/2$

$$S^2 = \hat{s}^m + i \hat{s}^n. \quad (4.5)$$

These fields have the correct charge to break the symmetry and generate the μ -term which has total \mathbf{Z}_N charge $-5\sigma - N/2$.

In a general supergravity theory [40, 41] the fermion mass matrix is

$$m_{ij}^\psi = m_{pl}^3 e^{G/2} (\nabla_i G_j + G_i G_j) \quad (4.6)$$

²Note section 2. The "i"s are not the same in S and z .

and the holomorphic components of the scalar mass matrix are

$$m_{ij}^{\phi^2} = m_{pl}^4 e^G (\nabla_i G_j + G^k \nabla_i \nabla_j G_k) \quad (4.7)$$

where $G = m_{pl}^{-2} K + \ln(m_{pl}^{-6} |W|^2)$ and subscripts on G denote derivatives with respect to the scalar fields ϕ_i or their conjugates ϕ_i^* . Respectively, (4.6) and (4.7) can be used to find μ

$$\mu = \langle m_{3/2} K_{H_u H_d} - F^{\bar{k}} K_{H_u H_d \bar{k}} \rangle \quad (4.8)$$

and $B\mu$

$$\begin{aligned} B\mu = & \langle 2m_{3/2}^2 K_{H_u H_d} - m_{3/2} F^{\bar{k}} K_{H_u H_d \bar{k}} + m_{3/2} F^m K_{H_u H_d m} \\ & - \left(m_{3/2} F^m K^{\bar{n}l} K_{\bar{l}m H_u} K_{n H_d} + (H_d \leftrightarrow H_u) \right) \\ & - F^n F^{\bar{m}} \left(\frac{1}{2} K_{H_u H_d n \bar{m}} - K^{j\bar{l}} K_{\bar{l}n H_u} K_{j\bar{m} H_d} + (H_d \leftrightarrow H_u) \right) \rangle \end{aligned} \quad (4.9)$$

where the indices run over the moduli fields and we have used that the superpotential does not contribute to either mass. Leading contributions come from Kahler potential terms

$$K \supset \alpha \frac{\langle S^1 \rangle^\dagger}{m_{pl}} H_u H_d + \beta \frac{\langle S^2 \rangle}{m_{pl}} H_u H_d + h.c. \quad (4.10)$$

where the coefficients α, β are expected to be $\mathcal{O}(1)$. Plugging the Kahler potential (4.10) into the formulas for μ and $B\mu$ gives the μ -term

$$\begin{aligned} \mu &= \alpha \frac{\langle S^1 \rangle}{m_{pl}} m_{3/2} + \alpha \frac{\langle F^{S^1} \rangle}{m_{pl}} \\ B\mu &= 2\alpha \frac{\langle S^1 \rangle}{m_{pl}} m_{3/2}^2 + \alpha \frac{\langle F^{S^1} \rangle}{m_{pl}} m_{3/2} + \beta \frac{\langle F^{S^2} \rangle}{m_{pl}} m_{3/2}. \end{aligned} \quad (4.11)$$

However, as a result of (4.2), (4.3) and the suppression of $m_{1/2}$ by order two orders of magnitude in the G_2 -MSSM, $\langle S^i \rangle m_{3/2} \simeq 10 \langle F^{S^i} \rangle$, the contribution to the masses coming from F -terms are sub-dominant, at least if we assume that $N \ll 10^4$. Therefore, to a good approximation

$$B\mu \simeq 2 \mu m_{3/2} \quad (4.12)$$

a fact which will have significant phenomenological consequences³.

The coefficients of the operators in (4.10) are in principle determined from M theory, but is not yet known how to calculate them precisely. It is natural to assume that the coupling coefficients are of $\mathcal{O}(1)$. When combined with a model of moduli stabilization, such as in the G_2 -MSSM described in [20, 32, 35] and briefly reviewed section (2.2), μ and $B\mu$ can be approximately determined. Since the real and imaginary components of the complex fields, S^1 (4.4) and S^2 (4.5), are expected to have similar, but not necessarily identical vevs, μ will generically have a phase, that will be unrelated to

³We leave the case of $N \geq 10^4$ for further study.

the phases that enter the gaugino masses. But, $B\mu$ and μ will have the same phase since both are proportional to S^1 and the same coupling constant.

Before moving on to the next section we discuss the possibility that other matter fields may be charged under the \mathbf{Z}_N symmetry, spontaneously break the \mathbf{Z}_N symmetry, and generate μ . Consider an $SU(5)$ -singlet matter field X that generates the μ -term via the superpotential coupling XH_uH_d . Since X is a matter field, M theory requires that it is charged under least one $U(1)$ symmetry. Then H_uH_d is not invariant under the $U(1)$, and consequently, the triplet mass term T_dT_u is not invariant, spoiling doublet-triplet splitting. Thus, such contributions should not occur.

Alternatively, the μ -term may be generated by a $U(1)$ invariant combination of two fields, for example by the operator

$$\frac{X_1X_2}{\Lambda}H_uH_d. \quad (4.13)$$

Requiring $\mu \gtrsim 10^3$ GeV, and taking $\Lambda \sim M_{GUT}$ this would require $\sqrt{\langle X_1X_2 \rangle} \gtrsim 10^{10}$ GeV. Radiative symmetry breaking will generally give a vev $\sim m_{3/2}$ —usually large vevs are associated with FI D -terms. But since FI D -terms are absent in M theory, it may be difficult for such large vevs to arise from here. The recent results of [42] do suggest that the F -term potential can generate large matter field vevs, however in that case the vevs are too large to be relevant for the μ problem. Therefore, we very tentatively conclude that a matter field spurion is not responsible for breaking the \mathbf{Z}_N symmetry and giving a physically relevant μ -term.

Finally, we comment on a potential domain wall problem. The moduli are stabilized away from a \mathbf{Z}_N point, which implies that the \mathbf{Z}_N symmetry was really only an approximate symmetry of the G_2 -manifold. The moduli stabilization serves to parameterize the amount that the G_2 -manifold differs from a \mathbf{Z}_N symmetric manifold. Therefore, since the \mathbf{Z}_N symmetry is not an exact symmetry of the G_2 manifold, the Lagrangian will explicitly break the \mathbf{Z}_N symmetry, and domains walls would not have formed in the early universe.

5 Origin of R -Parity in M theory

In the Standard Model, the Yukawa couplings and Higgs potential form the most general set of renormalizable couplings consistent with the gauge symmetries. In this sense, baryon (B) and lepton (L) number are accidental symmetries of the theory. However, this is not the case in supersymmetric

theories, which allow for the B and L violating renormalizable couplings⁴

$$W_{\mathcal{R}} = \lambda' LLe^c + \lambda'' LQd^c + \lambda''' u^c d^c d^c + \kappa Lh_u. \quad (5.2)$$

If the squark masses are not of order the GUT scale (which presumably they are not), these operators can lead to too rapid proton decay if not heavily suppressed. Hence one usually introduces R -parity, where the Standard Model fields have R -parity $+1$, while their supersymmetric partners have R -parity -1 . This forbids all the couplings in (5.2).

Additionally, R -parity invariance insures the stability of the LSP, and the absence of an R -parity can eliminate the LSP as a dark matter candidate. Therefore, in this section we will discuss the origin of R -parity in M theory, or at least an approximate R -parity that leaves the proton and LSP very long lived. Of course from a theoretical point of view an R -parity or equivalent symmetry should emerge from the theory and not be put in by hand.

The \mathbf{Z}_N symmetry constructed in Section 3 contains R -parity, but for generic moduli charges the complete \mathbf{Z}_N symmetry, including any R -parity subgroup, will be spontaneously broken. Although the \mathbf{Z}_N symmetry will prevent the superpotential couplings in (5.2) from being invariant, supersymmetry breaking will revitalize the operators just as in the case of the μ -term, from Kahler potential operators

$$K_{\mathcal{R}} \supset \frac{\tilde{S}^\dagger}{m_{pl}^2} LLe^c + \frac{\tilde{S}^\dagger}{m_{pl}^2} LQd^c + \frac{\tilde{S}^\dagger}{m_{pl}^2} u^c d^c d^c + \frac{\tilde{S}^\dagger}{m_{pl}} Lh_u \quad (5.3)$$

where the \tilde{S}^\dagger 's symbolically represent the moduli and need not all be the same.

Just as the μ -term was generated from the Kahler potential as a result of moduli stabilization, the effective superpotential can be calculated from the supersymmetry breaking contribution from (5.3) to

$$\begin{aligned} \lambda_{ijk} &\simeq m_{pl}^{-2} (\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}}) (K_{\mathcal{R}})_{ijk} \\ \kappa &\simeq m_{pl}^{-1} (\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}}) (K_{\mathcal{R}})_{Lh_u} \end{aligned} \quad (5.4)$$

for $\lambda = \lambda', \lambda'', \lambda'''$ and where i, j, k run over the matter fields. Comparing (5.4) to (4.11), one easily sees that $\kappa \sim \mu$, since both are generated the same way.

Then using that $\mu \sim \kappa$, the superpotential can be rewritten as

$$W_{\mathcal{R}} \simeq \frac{\mu}{m_{pl}} (LLe^c + LQd^c + u^c d^c d^c) + \mu Lh_u. \quad (5.5)$$

⁴ The final term in (5.2) can be rotated away in superpotential by a unitary transformation on (h_d, L) . This rotation will induce additional contributions to the lepton violating coupling constants λ' and λ'' that are proportional to the Yukawa couplings. Assuming that $\mu \gtrsim \kappa$, their sizes are approximately

$$\lambda' \sim y_e \frac{\kappa}{\mu} \quad \lambda'' \sim y_d \frac{\kappa}{\mu} \quad (5.1)$$

The trilinear couplings are suppressed but the lepton violating bilinear coupling is large and of order the μ -term—this is simply a consequence of κ not being suppressed. After rotating away the Lh_u term using the approximation (5.1), the superpotential simplifies to

$$W_{\mathcal{R}} \sim y_e L L e^c + y_d L Q d^c + \frac{\mu}{m_p} u^c d^c d^c \quad (5.6)$$

where smaller terms in $\lambda', \lambda'', \lambda'''$ have been dropped. Thus the lepton number violating trilinears pick up large contributions from the bilinear term, even if they were originally suppressed.

The proton lifetime for the decay mode $p \rightarrow e^+ \pi^0$ is estimated to be

$$\Gamma_{p \rightarrow e^+ \pi^0} \simeq \frac{\lambda''^2}{4\pi} \frac{\lambda'''^2}{4\pi} \frac{m_{\text{proton}}^5}{m_0^4}. \quad (5.7)$$

The current bounds on this partial decay width is $\tau_{p \rightarrow e^+ \pi^0} > 1.6 \times 10^{33}$ years [43]. For scalar masses in the G_2 -MSSM (~ 10 TeV see [35]) this gives the experimental bound

$$\lambda'' \lambda''' \lesssim 10^{-24} \quad (5.8)$$

which clearly excludes the superpotential (5.6), since $\lambda'' \sim y_e \sim 10^{-5}$ and $\lambda''' \sim \mu/m_{pl} \sim 10^{-14}$. Therefore, proton stability requires an additional form of R -parity invariance beyond the discrete symmetry proposed.

One possible way to preserve the R -parity is to simply assume that the G_2 -manifold in the vacuum is R -parity invariant, though not \mathbf{Z}_N invariant i.e. the vacuum partially breaks \mathbf{Z}_N to an R -parity subgroup. For example, take $N = 6$, then

	Coupling	\mathbf{Z}_6 charge
μ - term	$H_d H_u$	η^4
R-Parity	$M_{10} M_{\bar{5}} M_{\bar{5}}$	η^5
	$M_{\bar{5}} H_u$	η^3

(5.9)

for $\eta \equiv e^{i2\pi/6}$. If all moduli transform under the \mathbf{Z}_3 subgroup of \mathbf{Z}_6 , then \mathbf{Z}_6 is broken to \mathbf{Z}_2 R -Parity, since no R -parity couplings can be generated. This is technically satisfactory, but is presumably "non-generic". It could certainly emerge from M theory, but we will not consider it further here.

Alternatively, R -parity may manifest itself as matter-parity, a conserved remnant of a local, continuous $U(1)$ symmetry. As is well known, matter parity arises naturally in $SO(10)$ theories. When embedded into an $SO(10)$ unified theory, the Standard Model matter fields belong to a different representation than the Higgs fields— a generation of matter is contained in a **16** of $SO(10)$, while a pair of Higgs doublets comes from a **10** of $SO(10)$.

When $SO(10)$ is broken to $SU(5) \times U(1)_\chi$, for example by a discrete Wilson line, the Higgs fields and matter fields are charged differently under $U(1)_\chi$:

$$\begin{aligned} SO(10) &\rightarrow SU(5) \times U(1)_\chi \\ \mathbf{16} &\rightarrow \mathbf{10}_{-1} \oplus \bar{\mathbf{5}}_3 \oplus \mathbf{1}_{-5} \\ \mathbf{10} &\rightarrow \mathbf{5}_2 \oplus \bar{\mathbf{5}}_{-2}. \end{aligned} \quad (5.10)$$

where the subscript is the $U(1)_\chi$ charge.

The vacuum expectation values of the Higgses, which are contained in the $\mathbf{5}_2$ and $\bar{\mathbf{5}}_{-2}$ multiplets, will break the $U(1)_\chi$ symmetry into a discrete \mathbf{Z}_2 subgroup. This is because the Lagrangian is no longer invariant under the full local transformation $\Phi \rightarrow e^{i\alpha(x)q_d}\Phi$, but only the subgroup of transformations given by $\alpha(x) = \pi$. In terms of the $U(1)_\chi$ charges q_χ , the chiral multiplets have \mathbf{Z}_2 -parity $e^{i\pi q_\chi}$. Thus chiral superfields with even $U(1)_\chi$ charge will have parity $+1$ and fields with odd $U(1)_\chi$ charge will have parity -1 . The \mathbf{Z}_2 symmetry is exactly R -parity.

The only $SU(5)$ singlet with $U(1)_\chi$ charge is the $\mathbf{1}_{-5}$ field (and its conjugate), and thus this is the only field that can break $U(1)_\chi$ without breaking the SM gauge group. But since it has odd $U(1)_\chi$ charge, its vev will break R -parity. Therefore an $SO(10)$ completion of $U(1)_\chi$ will not contain an unbroken R -parity, but perhaps when combined with the \mathbf{Z}_N symmetry, R -parity violating operators may be sufficiently suppressed to allow a long lived proton and LSP. Next we estimate these lifetimes.

The singlet field $\mathbf{1}_{-5}$ can be considered to be the right-handed neutrino, ν^c , since it has the right quantum numbers to make the operator $\nu^c h_u L$ invariant under $U(1)_\chi$. However, if $\langle \nu^c \rangle \neq 0$, all baryon and lepton violating operators in (5.2) will be generated via the superpotential

$$W_R \sim \nu^c L L e^c + \nu^c L Q d^c + \nu^c u^c d^c d^c + \nu^c h_u L. \quad (5.11)$$

The operators in (5.11) should be suppressed and can be forbidden by the \mathbf{Z}_N symmetry. The story will be the same as above and the Kahler potential operators will generate (5.11), but with additional suppression coming from $U(1)_\chi$ breaking

$$W_R = \left(\frac{\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}}}{m_{pl}^2} \right) \left(\frac{\langle \nu^c \rangle}{m_p} \right) (L L e^c + L Q d^c + u^c d^c d^c) + \left(\frac{\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}}}{m_{pl}} \right) \left(\frac{\langle \nu^c \rangle}{m_p} \right) L h_u. \quad (5.12)$$

Diagonalizing away the $L h_u$ term, and using (4.11) gives

$$W_R \sim y_e \frac{\langle \nu^c \rangle}{m_p} L L e^c + y_d \frac{\langle \nu^c \rangle}{m_p} L Q d^c + \frac{\mu}{m_p} \frac{\langle \nu^c \rangle}{m_p} u^c d^c d^c. \quad (5.13)$$

where again large lepton violating trilinear terms are induced by the rotation.

To be conservative in our estimates, we can take $\langle \nu^c \rangle \sim \text{TeV}$, which may be expected from radiative symmetry breaking [44]. In this limit, proton decay constraints are safe from R -parity

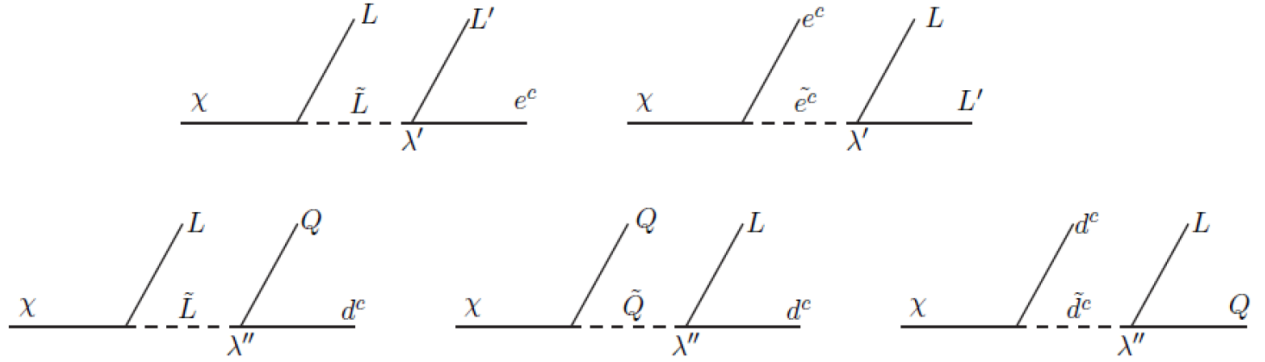


Figure 1: Decays of the LSP. Only the lepton number violating diagrams are shown, since the lepton number violating couplings– λ' (in the first line) and λ'' (in the second line)– receive large contributions (compared to the baryon number violating couplings) when the bilinear R -parity violating term, $h_u L$, is rotated away. Primes on the L indicate that the lepton flavor is different than the slepton flavor. Figures from [1].

violation, but there are more stringent constraints coming from the LSP lifetime. Current bounds on the LSP lifetime are slightly model dependent, but for the most part are [45]

$$\tau_{LSP} \lesssim 1 \text{ sec} \quad \text{OR} \quad \tau_{LSP} \gtrsim 10^{25} \text{ sec} . \quad (5.14)$$

The first bound excludes the region where the LSP decays would ruin the successful predictions of big bang nucleosynthesis on light nuclei abundances [46, 47]. The other region is excluded by indirect dark matter detection experiments that search for energetic positrons and anti-protons coming from decaying or annihilating relics [48–51].

The LSP lifetime can be calculated in terms of the general R -parity violating superpotential couplings (5.2). Diagrams in Figure 1 lead to an LSP lifetime

$$\tau \approx \frac{10^{-17} \text{ sec}}{\lambda^2} \left(\frac{m_0}{\text{TeV}} \right)^4 \left(\frac{100 \text{ GeV}}{m_{LSP}} \right)^5 \quad (5.15)$$

where $\lambda = \lambda', \lambda'', \lambda'''$ and m_0 is the mass of the sfermion mediating the decay. Taking $\lambda = \frac{\langle \nu^c \rangle}{m_p} \sim 10^{-15}$, $m_0 \sim 10 \text{ TeV}$, and $m_{LSP} \sim 100 \text{ GeV}$ gives

$$\tau_{LSP} \sim 10^{17} \text{ sec} , \quad (5.16)$$

about the age of the universe. The R -parity violating couplings still need to be about $10^{-4} \sim 10^{-5}$ smaller to have an LSP lifetime greater than 10^{25} seconds.

There are several ways additional suppressions might arise. We have not yet discussed the possibility of there being a horizontal family structure to the couplings. This could appear as a

Froggett-Nielson symmetry, or a symmetry relating the locations of the matter singularities on the G_2 manifold, and would be responsible for forging the quark and lepton hierarchy. It may also suppress the LSP decay width pass the astrophysical bounds. Family symmetries arise naturally from the E_8 structure [52], which can also explain why the Standard Model has three generation, and this may hint towards a larger gauge theory. We leave this issue to future work..

If the family symmetry is not the answer, then it may be the case that resolution of the E_8 singularity to $SU(5)$ preserves a $U(1)$ symmetry—whose charges are necessarily given as a linear combination of four $U(1)$ s belonging to the coset group $E_8/SU(5)$ —and is broken to an exactly conserved R -parity. There are two well known examples, $U(1)_\chi$ and $U(1)_\psi$, defined as the symmetries coming from the breaking $SO(10) \rightarrow SU(5) \times U(1)_\chi$ and $E_6 \rightarrow SO(10) \times U(1)_\psi$. However, $U(1)_\chi$ does not contain a field that can break $U(1)_\chi$ to R -Parity, and $U(1)_\psi$ forbids Higgs triplet masses, spoiling doublet-triplet splitting, so neither of these choices give a conserved R -parity.

However, there is a possibility that $U(1)$ symmetry is similar to $U(1)_\chi$, in that the MSSM fields and right handed neutrinos have the same charge assignment as in $U(1)_\chi$, but has additional $SU(5)$ singlet fields with even charges⁵. These theories can then be broken to a *conserved* R -parity, when the additional singlets get vevs. It is easy to construct such a linear combination, though it is unclear why from a purely theoretical perspective why G_2 compactifications would favor this $U(1)$ symmetry. For instance, if $U(1)_a \times U(1)_b$ is the cartan subgroup of $SU(3)$ in the breaking pattern $E_8 \rightarrow E_6 \times SU(3)$, then the $U(1)$ given by the linear combination of charges

$$q_\chi + 5(q_a - q_b)$$

allows for conical singularities that give rise to MSSM and right handed neutrino fields with $U(1)_\chi$ charges, but with additional $SU(5)$ singlets with charges $q = \pm 10$. The vevs of the additional singlets will break the $U(1)$ to a Z_{10} symmetry that contains a Z_2 R -parity.

Finally we note (for the non string duality oriented reader) that $E_8 \times E_8$ is well motivated theoretically if the G_2 -manifold is a $K3$ fibration. This is because the intersection matrix of 2-cycles inside $K3$ contain the Cartan matrix of $E_8 \times E_8$. It is in this case—that the gauge-theory of M theory matches the gauge theory of $E_8 \times E_8$ Heterotic string theory— in which M theory on a $K3$ -fibered G_2 -manifold and the heterotic string theory on a T^3 - fibered Calabi-Yau threefold are dual.

To summarize, we find that incorporating the μ parameter into the structure of M theory compactified on a G_2 -manifold, with stabilized moduli, can lead to a broken discrete symmetry allowing μ to be non-zero. R -parity is slightly broken, giving an LSP lifetime long enough to be the dark

⁵If this $U(1)$ symmetry is to be broken to R -parity, then requiring the symmetry to be flavor blind, allowing for Higgs triplet masses, and allowing an explanation for neutrino masses, basically constrains the charges of the MSSM and right handed neutrino fields to be the $U(1)_\chi$ charges.

matter, but not quite long enough to evade satellite detector constraints. The theoretical structure allows for family symmetries, or an embedding of R -parity into E_8 , both of which stabilize the LSP lifetime to be consistent with the experimental constraints. An example of the latter case is given above, so this is indeed a possibility. Either case will lead to the same dark matter phenomenology. The R -parity completion of this story is an interesting avenue for further investigation.

6 Phenomenology

The M theory framework, along with moduli stabilization in the G_2 -MSSM, allows one to estimate the high-scale SUSY breaking masses and μ to within a factor of a few. This allows M theory to make many phenomenological predictions. For some cases even small variations in the high-scale theory can have significant phenomenological consequences. In particular, the low-scale values of μ and $\tan\beta$ have significant implications for dark matter properties, and thus it is crucial to have a good understanding of their low-scale values while considering the M theory predictions of the high-scale masses.

6.1 Electroweak Symmetry Breaking

The first and foremost phenomenological constraint is that the theory accurately produce electroweak symmetry breaking (EWSB). That is, the theory must give a stable potential (bounded from below), break the electroweak symmetry and allow for the correct Z-boson mass. Respectively, these three conditions can be quantified by the following tree level constraints at the EWSB scale

$$\begin{aligned} |B\mu| &\leq \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + |\mu|^2 \\ |B\mu|^2 &\geq (m_{H_u}^2 + |\mu|^2)(m_{H_d}^2 + |\mu|^2) \\ M_Z^2 &= -2|\mu|^2 + 2\frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \end{aligned} \tag{6.1}$$

where $\tan\beta$ is not an independent parameter, but is determined by

$$\sin 2\beta = \frac{-2B\mu}{m_A^2}. \tag{6.2}$$

and

$$m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2. \tag{6.3}$$

where A is the pseudoscalar Higgs boson.

To get a feeling for $\tan\beta$, we plug in the expected values (at the unification scale and with degenerate scalars) of $B\mu \simeq 0.2m_{3/2}^2$ and $m_A^2 \simeq 2m_{3/2}^2$, into (6.2) which gives $\tan\beta \simeq 10$. On

the other hand, RGE flow will lower the values of both $B\mu$ and m_A^2 , resulting in variations around $\tan\beta \simeq 10$. In Section 7.1.1, a numerical scan will show a lower bound of $\tan\beta \gtrsim 5$, when scalars are taken to be degenerate at the unification scale.

The lowest values of $\tan\beta$ occur for the smallest values of m_A^2 . The EW scale value for the mass depends on the running of the Higgs scalar masses, and in turn is very sensitive to the values of the squark masses. For specific non-degenerate values of scalar masses at the unification scale, m_A^2 can be of order $B\mu$ at the EW scale, resulting in values of $\tan\beta < 5$. We will consider this situation in Section 7.1.2.

At tree level the mass of the Z-boson is determined by the four Higgs parameters

$$M_Z(m_{H_u}^2, m_{H_d}^2, |\mu|^2, \tan\beta). \quad (6.4)$$

These parameters not only depend on their respective values at the high-scale, but also on other masses as a result of RGE-flow. Assuming that the scalar masses are much larger than the gaugino masses, M_Z has strongest dependence on the Higgs mass parameters and stop masses

$$M_Z(\hat{m}_{H_u}^2, \hat{m}_{H_d}^2, \hat{B}\mu, |\hat{\mu}|^2, \hat{m}_{Q_3}^2, \hat{m}_{U_3}^2). \quad (6.5)$$

where hatted ($\hat{}$) masses refer to GUT scale values.

Interestingly the cancellation between the soft scalars masses contributing to M_Z can be significant, even in the case in which the scalar masses are unified at the GUT scale

$$\hat{m}_{H_u}^2 = \hat{m}_{H_d}^2 = \hat{m}_{Q_3}^2 = \hat{m}_{U_3}^2.$$

Naively what one thought was a large fine-tuning between the Higgs soft-masses and μ in eq. (6.1) for M_Z , is in fact smaller. This is evident (see Figure 2) from the fact that the scalar masses can be of order the gravitino mass at unification and μ can be an order of magnitude smaller, but the cancellation in eq. (6.1) for M_Z still occurs. In this sense, the ratio $\mu/m_{3/2}$, shown in Figure 2, might be considered a measure of the fine-tuning involved in EWSB. In other words, the smaller the ratio, the less fine tuning there will be of μ against the scalar masses in order to have the correct value for M_Z .

6.1.1 Degenerate Scalars

A numerical scan was performed over M theory parameter space described in [35] using SOFT-SUSY [53]⁶. We allow for the following variation in the G_2 -MSSM parameters,

- $10 \text{ TeV} \leq m_{3/2} \leq 20 \text{ TeV}$ – the gravitino mass

⁶See [54] for general phenomenological discussions.

- $10 \leq V_7 \leq 40$ – the volume of the G_2 -manifold in units of the eleven-dimensional Planck length
- $-10 \leq \delta \leq 0$ – the size of the threshold corrections to the (unified) gauge coupling, α_{GUT}^{-1} .⁷

An interested reader is referred to Section V of [35] for variations in the spectra of G_2 -MSSM models. In addition, order one variations are allowed for the coefficients in (4.11) for the formula for μ , while it is imposed that $B\mu$ is in the range.

$$1 \mu m_{3/2} < B\mu < 3 \mu m_{3/2}. \quad (6.6)$$

The results are shown in Figure 2. As is evident from the plot, values of μ much smaller than the gravitino mass are allowed under all the constraints, signaling a non-imposed cancellation among the scalars contributing to M_Z . Of note is the fact that $\tan\beta$ and μ are inversely correlated, which will play a significant role in limiting the maximum spin-independent scattering cross-section, when scalar masses are unified at the high scale.

6.1.2 Non-Degenerate Scalars and Low $\tan\beta$

We also consider the possibility that M theory allows for scalar unification to be somewhat perturbed (at the factor of two to three level). Since eventually we will be interested in calculating the largest possible spin-independent scattering cross sections we will only consider high-scale scalar masses that give $\tan\beta \lesssim 3$ —since the scattering cross sections decrease with increasing $\tan\beta$.

Consider the 1-Loop RGE equations, where only terms proportional to λ_t are kept, while neglecting the λ_t running. The RGE equations of the relevant scalars are:

$$\begin{aligned} 8\pi^2 \frac{dm_{H_u}^2}{dt} &= 3|\lambda_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2) \\ 8\pi^2 \frac{dm_{U_3}^2}{dt} &= 2|\lambda_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2) \\ 8\pi^2 \frac{dm_{Q_3}^2}{dt} &= 1|\lambda_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2) \\ 8\pi^2 \frac{dA_t^2}{dt} &= 6\lambda_t^2 A_t \end{aligned}$$

whose solution is

$$\begin{aligned} m_{H_u}^2 &= \frac{1}{2} \left(\hat{m}_{H_u}^2 - \hat{m}_{U_3}^2 - \hat{m}_{Q_3}^2 + e^{\frac{3t\lambda^2}{4\pi^2}} (|\hat{A}_t|^2 (-1 + e^{\frac{3t\lambda^2}{4\pi^2}}) + \hat{m}_{H_u}^2 + \hat{m}_{U_3}^2 + \hat{m}_{Q_3}^2) \right) \\ m_{U_3}^2 &= \frac{1}{3} \left(-\hat{m}_{H_u}^2 + 2\hat{m}_{U_3}^2 - \hat{m}_{Q_3}^2 + e^{\frac{3t\lambda^2}{4\pi^2}} (|\hat{A}_t|^2 (-1 + e^{\frac{3t\lambda^2}{4\pi^2}}) + \hat{m}_{H_u}^2 + \hat{m}_{U_3}^2 + \hat{m}_{Q_3}^2) \right) \\ m_{Q_3}^2 &= \frac{1}{2} \left(-\hat{m}_{H_u}^2 - \hat{m}_{U_3}^2 + 5\hat{m}_{Q_3}^2 + e^{\frac{3t\lambda^2}{4\pi^2}} (|\hat{A}_t|^2 (-1 + e^{\frac{3t\lambda^2}{4\pi^2}}) + \hat{m}_{H_u}^2 + \hat{m}_{U_3}^2 + \hat{m}_{Q_3}^2) \right) \\ A_t^2 &= \hat{A}_t^2 e^{\frac{3t\lambda^2}{8\pi^2}} \end{aligned} \quad (6.7)$$

⁷see Section IV of [35] for the precise definition of δ

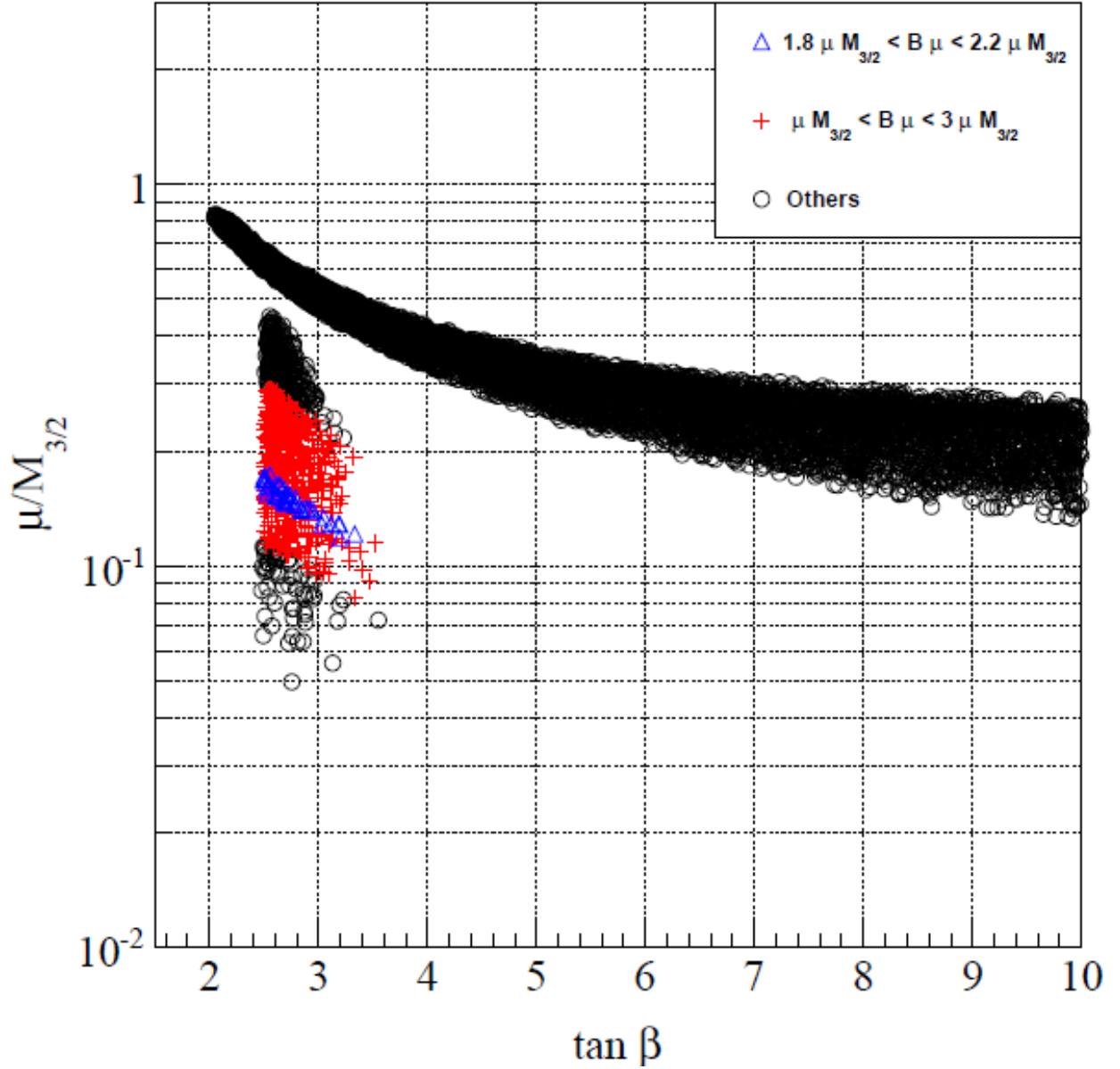


Figure 2: $\mu/m_{3/2}$ vs. $\tan\beta$. The upper band scans over the G_2 -MSSM parameter space with *degenerate scalars* at the unification scale. The lower region on the left (low $\tan\beta$) scans over the G_2 -MSSM parameter space where the scalar mass ratio $\hat{m}_{H_u}^2 : \hat{m}_{U_3}^2 : \hat{m}_{Q_3}^2 = 3 : 2 : 1$ is required to be accurate within 20%. The black points show models that correctly break the EW symmetry, but are inconsistent with constraint $1 \mu m_{3/2} < B\mu < 3 \mu m_{3/2}$, so we expect them to not be valid solutions. The red points satisfy the constraint on $B\mu$ as given in the legend. The empty space on the plot, between the two regions, is expected to be filled in with complete scan over possible non-degenerate scalar mass parameter space. All points have EWSB.

where hatted () masses indicate GUT scale mass.

Since $m_{H_d}^2$ barely runs for low $\tan\beta$ and it is predicted that $\hat{\mu}^2$ is over an order of magnitude smaller than $m_{H_d}^2$, the cancellation in M_Z (6.1) should occur between $m_{H_u}^2$ and $m_{H_d}^2$. Therefore, $m_{H_u}^2$ needs to stay positive at the EWSB scale. Ignoring the exponentially suppressed terms in (6.7), we see that there are no choices of $\{\hat{m}_{H_u}^2, \hat{m}_{Q_3}^2, \hat{m}_{U_3}^2\}$ that leave all low-scale masses positive. On the other hand, there is a fixed point solution to the above RGEs

$$\hat{m}_{H_u}^2 : \hat{m}_{U_3}^2 : \hat{m}_{Q_3}^2 = 3 : 2 : 1 \quad (6.8)$$

where the non-exponentially suppressed terms are identically zero, insuring that if the trilinears are of order the scalars as expected in the G_2 -MSSM, all three masses will stay positive. This fixed point is analogous to the focus point solution in minimal supergravity (mSUGRA) theories [55, 56], as it minimizes the fine tuning of EWSB. However, unlike the focus point region of mSUGRA where the Higgs scalars run small due to RGE flow, here the scalars remain heavy, and are close to the gravitino mass.

Near this region, low $\tan\beta$ parameter space with EWSB can be realized. Results on the numerical scan can be seen in Figure 2.

6.2 The Nature of the LSP

As explained in detail in [35], the G_2 -MSSM framework gives rise to mostly Wino LSPs (as opposed to Bino LSPs). The tree level gaugino masses are degenerate at the GUT scale, but are suppressed by F -terms of the moduli relative to the gravitino mass to be of order the gaugino masses from the anomaly mediation contribution. The additional contribution from the anomaly lifts M_1 over M_2 , leading to mostly Wino LSP models. In the original G_2 -MSSM scenario, where it was simply that $\mu \sim m_{3/2}$, there were additional contributions to the gaugino masses from supersymmetric Higgs loops, proportional to μ [57], that for some choices of high scale parameters, could re-lift M_2 over M_1 . These models are disfavored by precision gauge coupling unification [35], and occur less frequently in parameter space here than the original models since $\mu \not\sim m_{3/2}$. However, smaller μ will tend to introduce a small Higgsino admixture into the mostly Wino LSP - a fact which has significant implications on dark matter discovery (Section 6.3). All these considerations combine to strongly suggest that a Wino-like LSP with mass $\sim 140 - 200$ GeV constitutes a significant fraction of the dark matter.

As emphasized in [58, 59], in order to obtain about the right relic density from the moduli decays, the LSP must be a Wino-like particle, with a large annihilation cross section of about $3 \times 10^{24} \text{ cm}^2$. A non-thermal history dominated by moduli and a wino LSP give a consistent picture for dark matter from the compactified string theory. Also encouraging is the fact that the PAMELA satellite data on

positrons and antiprotons can be consistently described by a Wino LSP [60–65]. More recently, by also considering Wino annihilations into photons and Z-bosons one finds a cross-section of about 10^{-26} cm^2 – a fact relevant for future Fermi data.

6.3 Direct Detection of Dark Matter

In December 2009, CDMS reported at most two possible WIMP candidate events, with a high likelihood of being background [66]. Combining with their previous data, this amounts to a bound on the spin-independent scattering cross-section of $\sigma_{si} \lesssim 6 \times 10^{-44} \text{ cm}^2$ for a WIMP of mass around 200 GeV. More recently, the XEXON100 experiment [67] reported observing no events after their first 11 days of running, slightly strengthening the CDMS bound. In the near future, XEXON100 is expected to report results that will probe much smaller scattering cross sections $\sigma_{SI} \sim 2 \times 10^{-45} \text{ cm}^2$. We will see that even this region is out of reach given the M theory predictions we calculate.

In the decoupling limit, defined when the pseudoscalar mass is much larger than the Z-boson mass, $m_{A^0} \gg M_Z$, the charged and heavy CP-even Higgses are also heavy, $m_{H^\pm} \simeq m_{H^0} \simeq m_{A^0}$. The other Higgs boson h^0 remains light and behaves in the same way as the SM Higgs boson. The lower bound on its mass, corresponds to the same bound on the SM Higgs boson, namely 114 GeV⁸ [68]. All the models consistent with all the theoretical and phenomenological constraints have light Higgs mass close to this LEP limit. Since the squarks are also heavy in G_2 -MSSM, the light Higgs boson exchange will give the only substantial contribution to the spin-independent scattering cross sections. The scattering of the LSP off nuclei is via the Higgsino component. While the LSP will be mostly Wino-like, the prediction that μ is of order the TeV scale implies that the LSP wavefunction can have non-trivial Higgsino mixing.

Following [69] we estimate the size of the direct detection cross section in the decoupling limit to be

$$\sigma_{SI}(\chi N \rightarrow \chi N) \approx 5 \times 10^{-45} \text{ cm}^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{Z_{H_u} \sin \beta - Z_{H_d} \cos \beta}{0.1} \right)^2 (Z_W - \tan \theta_W Z_B)^2 \quad (6.9)$$

where the Z 's give the composition of the LSP

$$\chi \equiv Z_B \tilde{B} + Z_W \tilde{W} + Z_{H_d} \tilde{H}_d + Z_{H_u} \tilde{H}_u. \quad (6.10)$$

This gives us an estimate of the largest direct detection scattering cross sections, which naively may seem that for $Z_{H_u} \sim 0.1$ can be very close to the reach of XENON. Eq. (7.1) can further be simplified, with the aid of analytical expressions for the neutralino mass matrix eigenvalues and

⁸Since there are theoretical and calculational uncertainties with calculating the Higgs mass, we will consider models with $m_h \geq 110 \text{ GeV}$.

eigenvectors [70–72]. Taking the limit $M_1 = M_2$, which maximizes the scattering cross section for fixed μ and $\tan\beta$, (7.1) becomes

$$\sigma_{\text{SI}}^{\text{MSSM}}(\chi N \rightarrow \chi N) \approx 6 \times 10^{-45} \text{cm}^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{1 \text{ TeV}}{\mu} \right)^2 \left(\frac{\sin 2\beta + M_2/\mu}{1 - (M_2/\mu)^2} \right)^2 \quad (6.11)$$

which falls off both with $\tan\beta$ and μ . Allowing for the variation in M_1 and M_2 in the G_2 -MSSM will only decrease this fraction. The value M_2/μ is typically around $.1 \sim .2$. The parameters for three different models, along with their scattering cross sections, can be seen in Table 1 and are appropriately labeled in Figure 3.

However, as shown in the previous section, when considering degenerate scalar masses at the unification scale EWSB imposes that small μ corresponds to large $\tan\beta$, and small $\tan\beta$ corresponds to large μ . Hence, large cross-sections, of order the XENON100 reach are not attainable for this region. To verify this we perform a scan of parameter space, using DarkSUSY [73]. The results are shown in Figure 3 where it is seen that the largest scattering cross-sections are $\sim 1 \times 10^{-45} \text{cm}^2$, close to, but slightly beyond the reach of XENON100.

In Figure 3 we also scan over the G_2 -MSSM parameter space, while requiring that the ratio $\hat{m}_{H_u}^2 : \hat{m}_{U_3}^2 : \hat{m}_{Q_3}^2 = 3 : 2 : 1$ be accurate within 20%. The spin-independent scattering cross-section reaches an upper limit of $1 \times 10^{-45} \text{cm}^2$, just beyond the XENON100 reach. Since this is the region where largest cross-sections appear, we can conclude that if the solution of the μ -problem proposed, along with moduli-stabilization in the G_2 -MSSM, is the model of nature, the XENON100 experiment will not observe a dark matter signal soon, but its next run and upgraded detectors may do so.

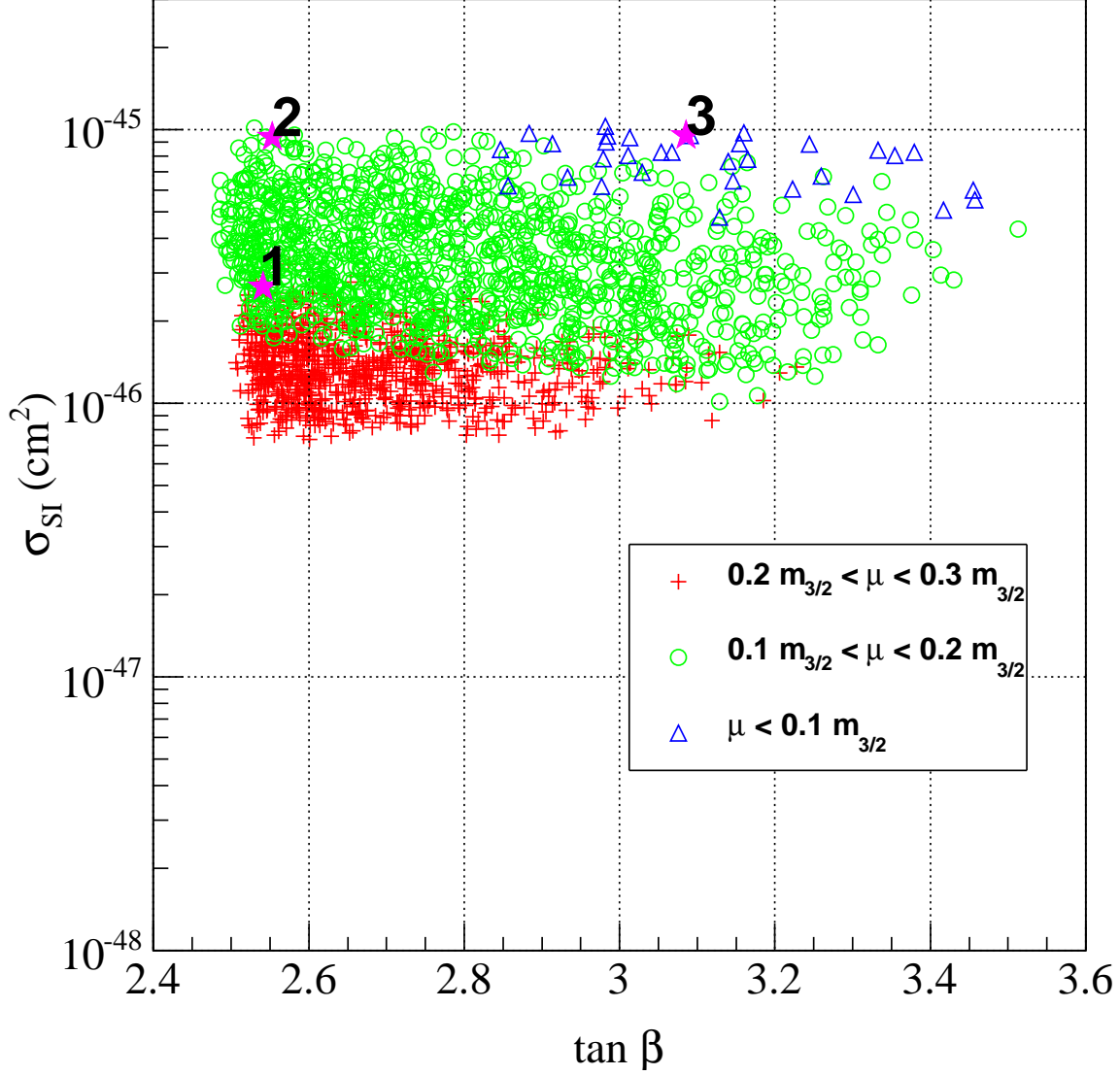


Figure 3: Spin-independent scattering cross-sections vs $\tan\beta$. The region shown scans over the G_2 -MSSM parameter space where the scalar mass ratio $\hat{m}_{H_u}^2 : \hat{m}_{U_3}^2 : \hat{m}_{Q_3}^2 = 3 : 2 : 1$ is required to be accurate within 20%. All points satisfy the constraint $\mu m_{3/2} < B\mu < 3\mu m_{3/2}$, have a SM-like Higgs with mass $m_h \geq 110$ GeV, and have EWSB. We also list the parameters for the 3 models in Table 1. In the region where EWSB, supergravity, and phenomenological constraints are satisfied, the upper limit on σ_{SI} is robust, but the lower limit can decrease if the sign of μ is reversed.

	Model 1	Model 2	Model 3
$M_{3/2}$	17.8 TeV	18.1 TeV	17.9 TeV
$\sqrt{B\mu_{\text{GUT}}}$	9.75 TeV	10.4 TeV	9.29 TeV
μ_{GUT}	3.79 TeV	2.10 TeV	1.69 TeV
M_1	151. GeV	153. GeV	150. GeV
M_2	145. GeV	143. GeV	138. GeV
μ	3.89 TeV	2.15 TeV	1.77 TeV
M_A	18.8 TeV	19.0 TeV	18.2 TeV
m_h	110. GeV	110. GeV	115. GeV
M_{χ_1}	141 GeV	143 GeV	141 GeV
M_{χ_2}	143 GeV	147 GeV	145 GeV
$M_{\chi_1^\pm}$	141 GeV	144 GeV	142 GeV
$Z_{\tilde{W}}$	0.94	0.91	0.91
$Z_{\tilde{B}}$	-0.35	-0.41	-0.41
$Z_{\tilde{H}_d}$	-0.02	-0.04	-0.05
$Z_{\tilde{H}_u}$	0.01	0.02	0.02
$\tan \beta$	2.53	2.37	2.87
$\sigma_{\text{SI}}[\text{cm}^2]$	$3. \times 10^{-46}$	$9. \times 10^{-46}$	$1. \times 10^{-45}$
$\sigma_{\text{SD}}[\text{cm}^2]$	$5. \times 10^{-45}$	$5. \times 10^{-44}$	$1. \times 10^{-43}$

Table 1: High scale and low scale parameters for 3 models with larger spin independent scattering cross sections. All models shown belong to the parameter space where the scalar mass ratio $\hat{m}_{H_u}^2 : \hat{m}_{U_3}^2 : \hat{m}_{Q_3}^2 = 3:2:1$ is accurate within 20%. We assume that details of the calculation and software outputs are sufficiently uncertain to allow $m_h \gtrsim 110$ GeV to be consistent with LEP bounds.

7 Conclusions

We have argued that if our universe is described by M theory compactified on a manifold of G_2 holonomy, with doublet-triplet splitting solved in the way originally proposed by Witten [19], then there is a simple solution to the μ -problem: strong coupling dynamics in the hidden sector will generate a non-perturbative potential for the moduli, which stabilizes all the moduli vevs, and breaks the symmetry forbidding μ . Then, following the numerical analysis done in the G_2 -MSSM [35], the breaking will generate $\mu \sim \langle \frac{S}{m_{pl}} \rangle m_{3/2} \sim 0.1 m_{3/2} \sim 2$ TeV. This then implies a non-zero Higgsino component of the mostly Wino LSP, with an upper limit, which in turn gives an upper limit of about $1 \times 10^{-45} \text{ cm}^2$ on the spin-independent scattering cross-section, somewhat below the reach of the XENON100 experiment, as well as a lower limit of about 10^{-46} cm^2 . The Wino-like LSP also can account for the PAMELA positron and antiproton excesses [62, 74], and gives about the desired relic density for a non-thermal cosmological history [33], as expected in theories with moduli.

Since the scalars are of order $m_{3/2} \gtrsim 20$ TeV, the Higgs sector is decoupled, and the light Higgs boson behaves like a Standard Model one. Its mass is predicted to be of order 110-120 GeV. If we insist on a good description of the Pamela data plus consistent compactification, we find an LSP mass from about 140-155 GeV, and an annihilation cross section $2 - 3.5 \times 10^{-24} \text{ cm}^3/\text{s}$. The annihilation to γ/Z ranges from $(0.7 - 1.2) \times 10^{-26}$.

Additionally, we noted that an exact R -parity could arise through ‘partial symmetry breaking’, though this isn’t obviously motivated by the theory itself. An alternative is that R -parity is either an exact remnant of a broken continuous gauge symmetry, or only an approximate symmetry of larger broken discrete and continuous groups. In either case, this requires the inclusion of additional $U(1)$ gauge symmetries, suggesting that the GUT group is larger than $SU(5)$, and may originate from an E_8 singularity.

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Appendix A: Largest Spin Independent Cross Sections

Following [69], the spin-independent cross section for the LSP scattering off a nucleon, is given in the decoupling limit ($M_Z \ll M_A$) by the approximation

$$\sigma_{\text{SI}}(\chi N \rightarrow \chi N) \approx 5 \times 10^{-45} \text{cm}^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{Z_{H_u} \sin \beta - Z_{H_d} \cos \beta}{0.1} \right)^2 (Z_W - \tan \theta_W Z_B)^2 \quad (7.1)$$

where the Z ’s give the composition of the LSP

$$\chi \equiv Z_B \tilde{B} + Z_W \tilde{W} + Z_{H_d} \tilde{H}_d + Z_{H_u} \tilde{H}_u. \quad (7.2)$$

Consider the neutralino mass matrix [75]:

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\ 0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\ -M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & -\mu \\ M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & -\mu & 0 \end{pmatrix} \quad (7.3)$$

in the $\{\tilde{B}, \tilde{W}, \tilde{H}_d, \tilde{H}_u\}$ basis. The analytical expression [70–72] for the components in the LSP can be written as:

$$\begin{aligned} \alpha Z_B = z_B &= -\sin \theta_W \\ \alpha Z_W = z_W &= \cos \theta_W \frac{M_1 - M_\chi}{M_2 - M_\chi} = \cos \theta_W \frac{(M_1 - M_\chi)^2}{\Delta} \end{aligned}$$

$$\begin{aligned}
\alpha Z_{H_d} = z_{H_d} &= \frac{\mu (M_1 - M_\chi) (M_2 - M_\chi) + M_Z^2 \sin \beta \cos \beta ((M_1 - M_2) \cos^2 \theta_W + M_2 - M_\chi)}{M_Z (M_2 - M_\chi) (-\mu \cos \beta + M_\chi \sin \beta)} \\
\alpha Z_{H_u} = z_{H_u} &= \frac{M_\chi (M_1 - M_\chi) (M_2 - M_\chi) + M_Z^2 \cos^2 \beta ((M_1 - M_2) \cos^2 \theta_W + M_2 - M_\chi)}{M_Z (M_2 - M_\chi) (-\mu \cos \beta + M_\chi \sin \beta)}
\end{aligned} \tag{7.4}$$

where $\alpha = \sqrt{z_B^2 + z_W^2 + z_{H_d}^2 + z_{H_u}^2}$ is an overall normalization factor and $\Delta \equiv (M_\chi - M_1)(M_\chi - M_2)$.

The combination $Z_{H_u} \sin \beta - Z_{H_d} \cos \beta$, that appears in the scattering cross section takes an especially simple form

$$Z_{H_u} \sin \beta - Z_{H_d} \cos \beta = \frac{(M_\chi \sin \beta - \mu \cos \beta) (M_1 - M_\chi) (M_2 - M_\chi)}{M_Z (M_2 - M_\chi) (-\mu \cos \beta + M_\chi \sin \beta)} = \frac{M_1 - M_\chi}{M_Z}. \tag{7.5}$$

It is clear from (7.5) that as $M_1 - M_\chi$ increases, $Z_{H_u} \sin \beta - Z_{H_d} \cos \beta$ grows slower than the Z_W component. Thus after normalization both the \tilde{H}_u and the \tilde{H}_d components will decrease. So the maximum of $Z_{H_u} \sin \beta - Z_{H_d} \cos \beta$ is realized when $M_1 - M_\chi$ is minimal.

The eigenvalues of the neutralino mass matrix (7.3) are given by the solutions to:

$$(x - M_1) (x - M_2) (x - \mu) (x + \mu) + (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta) M_Z^2 \mu \sin 2\beta = 0 \tag{7.6}$$

Then the LSP mass, corresponding to $M_\chi \equiv x$, can be found by taking the limit $M_\chi \ll \mu$, so that (7.6) is simply a quadratic equation. Then it is easy to see that the minimal value of $M_1 - M_\chi$, which maximizes $Z_{H_u} \sin \beta - Z_{H_d} \cos \beta$, corresponds to the situation when $M_1 - M_2$ is also minimized. Additionally, when $M_1 = M_2$, the term $Z_W - \tan \theta_W Z_B$ also reaches its maximum. Thus the maximum scattering cross sections will occur when $M_1 = M_2$.

To normalize the expressions in (7.4) (i.e. finding α) is tedious. Instead, a new basis is defined where $\tilde{\gamma} = \cos \theta_W \tilde{B} + \sin \theta_W \tilde{W}$ and $\tilde{Z} = -\sin \theta_W \tilde{B} + \cos \theta_W \tilde{W}$, where in the supersymmetric limit, these are the superpartners of the photon and Z -boson, respectively. The new mass matrix, in the $\{\tilde{\gamma}, \tilde{Z}, \tilde{H}_d, \tilde{H}_u\}$ is

$$\mathcal{M} = \begin{pmatrix} M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W & (M_2 - M_1) \sin \theta_W \cos \theta_W & 0 & 0 \\ (M_2 - M_1) \sin \theta_W \cos \theta_W & M_2 \cos^2 \theta_W + M_1 \sin^2 \theta_W & M_Z \cos \beta & -M_Z \sin \beta \\ 0 & M_Z \cos \beta & 0 & -\mu \\ 0 & -M_Z \sin \beta & -\mu & 0 \end{pmatrix} \tag{7.7}$$

Taking the limit $M_1 = M_2 \equiv M$, one immediately finds that $\tilde{\gamma}$ is an eigenvector with mass eigenvalue M . The next lightest eigenvector of the remaining 3×3 sub-matrix will be mostly \tilde{Z} , and to leading order in M_Z/μ , the mass is

$$M_\chi \simeq M - \frac{M_Z^2}{\mu} \left(\frac{M}{M_Z} - \sin 2\beta \right) \tag{7.8}$$

Next we will assume that the phases of M and μ are such that absolute value of M_χ is smaller than $|M|$, so that it is indeed the LSP. The other scenario, in which the LSP is mostly $\tilde{\gamma}$, will have negligible scattering cross-section.

Diagonalizing the remaining 3×3 sub-matrix, the coefficients of \tilde{H}_u and \tilde{H}_d component to leading order is

$$\begin{aligned} Z_{H_d} &= \frac{1}{2} \left(\frac{(\sin \beta - \cos \beta) M_Z}{\mu + M} + \frac{(\sin \beta + \cos \beta) M_Z}{\mu - M} \right) = \frac{M_Z (\mu \sin \beta + M \cos \beta)}{\mu^2 - M^2} \\ Z_{H_u} &= \frac{1}{2} \left(\frac{(\sin \beta - \cos \beta) M_Z}{\mu + M} - \frac{(\sin \beta + \cos \beta) M_Z}{\mu - M} \right) = -\frac{M_Z (\mu \cos \beta + M \sin \beta)}{\mu^2 - M^2} \end{aligned} \quad (7.9)$$

and from the definition on \tilde{Z} ,

$$Z_W - \tan \theta_W Z_B = \cos \theta_W^{-1}. \quad (7.10)$$

Finally, using (7.9) and (7.10) as inputs to (7.1) the upper limit for the cross section is

$$\sigma_{\text{SI}} (\chi N \rightarrow \chi N) \approx 6 \times 10^{-45} \text{cm}^2 \left(\frac{115 \text{ GeV}}{m_h} \right)^4 \left(\frac{1 \text{ TeV}}{\mu} \right)^2 \left(\frac{\sin 2\beta + M_2/\mu}{1 - (M_2/\mu)^2} \right)^2 \quad (7.11)$$

From the discussion in the text we expect $M_2/\mu \lesssim 0.2$, $\sin 2\beta \lesssim 0.8$ and $\mu \gtrsim 1 \text{ TeV}$, giving largest scattering cross-sections around $\sigma_{\text{SI}} \lesssim 6 \times 10^{-45} \text{cm}^2$. However, as discussed in Section 6.3, the constraints cannot all be satisfied simultaneously, so in practice only a cross section of about 10^{-45}cm^2 could be achieved.

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